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SOLVING AN EOQ MODEL IN AN INVENTORY PROBLEM BY USING OCTAGONAL FUZZY NUMBERS

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ABSTRACT

In this paper, a general fuzzy inventory problem is discussed. There are several approaches by different authors to solve such an inventory problem. We introduce octagonal fuzzy numbers by which we develop a new model to solve the problem. Ordering cost, holding cost and order quantity are taken as octagonal fuzzy numbers. Signed Distance method is used for defuzzification. The proposed model is illustrated through numerical examples.

KEYWORDS: Inventory Problem, Octagonal Fuzzy Numbers, Signed Distance Method

INTRODUCTION

In the past few decades, inventory problems have been widely studied by researchers. In the crisp inventory models, all the parameters in the total cost are known and have definite values. But in the practical situation it is not possible. Hence fuzzy inventory models fulfill that gap. Different fuzzy inventory models occur due to various fuzzy cost parameters in the total cost. Fuzzy mathematical programming has been applied to several fields like project network, reliability, optimization, transportation, media selection for advertising, air pollution etc., The first quantitative treatment of inventory was the simple EOQ model. The fuzzy set theory was first introduced by Zadeh [3], and has now been applied in inventory control systems to model behavior more realistically. In 1981, Sommer[4] used fuzzy dynamic programming to solve a real-world inventory and production scheduling problem. The model was developed by Harris [5], Wilson [6] and they investigated about stock control and optimum costs. Many applications of fuzzy set theory can be found in Zimmerman [7].

The model developed by S. U. Malini and Felbin C. Kennedy (8) that they introduced octagonal fuzzy numbers for transportation problems. Dutta [9] developed the fuzzy inventory problem using trapezoidal fuzzy numbers.

Several fuzzy inventory models are developed by different researchers. There is no fuzzy inventory model by the use of octagonal fuzzy numbers true to our knowledge. Our Endeavour is to develop an innovative model using octagonal fuzzy numbers. In our proposed model we consider ordering cost, holding cost and order quantity as octagonal fuzzy numbers. Signed distance method is used for defuzzification. Numerical examples are given to illustrate this new model.

Fuzzy Numbers

Any fuzzy subset of the real line R, whose membership function μ_A satisfy the following conditions, is a generalized fuzzy number $\tilde{A}$.

- μ_A is a continuous mapping from R to the closed interval [0, 1],
- $\mu_A = 0, \ -\infty < x \le a_1$
- $\mu_A = L(x)$ is strictly increasing on $[a_1, a_2]$
- $\mu_A = w_A, \ a_2 \le x \le a_3$
- $\mu_A = R(x)$ is strictly decreasing on $[a_3, a_4]$
- $\mu_A = 0, \ a_4 \le x \le \infty$
- Where $0 < w_A \le 1$ and a_1, a_2, a_3 and a_4 are real numbers. Also this type of generalized fuzzy number is denoted as $\tilde{A} = (a_1, a_2, a_3, a_4; w_A)_{LR}$; when $w_A = 1$, it can be simplified as $\tilde{A} = (a_1, a_2, a_3, a_4; w_A)_{LR}$.

Octagonal Fuzzy Numbers

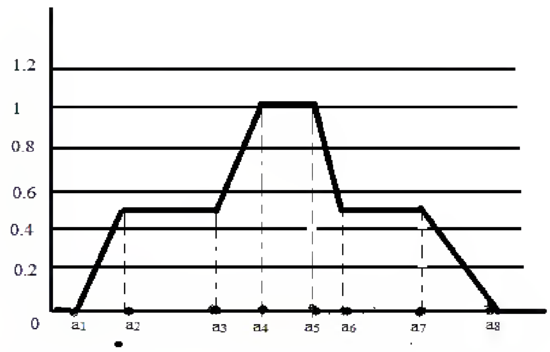
A fuzzy number $\tilde{A}$ is a normal octagonal fuzzy number denoted by $(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$ where $a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8$ are real numbers and its membership function $\mu_{\tilde{A}}(x)$ is given below

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & \text{for } x < a_1 \\ k\left(\dfrac{x - a_1}{a_2 - a_1}\right) & \text{for } a_1 \le x \le a_2 \\ k & \text{for } a_2 \le x \le a_3 \\ k + (1-k)\left(\dfrac{x - a_3}{a_4 - a_3}\right) & \text{for } a_3 \le x \le a_4 \\ 1 & \text{for } a_4 \le x \le a_5 \\ k + (1-k)\left(\dfrac{a_6 - x}{a_6 - a_5}\right) & \text{for } a_5 \le x \le a_6 \\ k & \text{for } a_6 \le x \le a_7 \\ k\left(\dfrac{a_8 - x}{a_8 - a_7}\right) & \text{for } a_7 \le x \le a_8 \\ 0 & \text{for } x \ge a_8 \end{cases}$$

where $0 < k < 1$.

Remark: If $k = 0$, the octagonal fuzzy number reduces to the trapezoidal number (a_3, a_4, a_5, a_6) and if $k = 1$, it reduces to the trapezoidal number (a_1, a_4, a_5, a_8).

Graphical Representation of a normal octagonal fuzzy number for $k = 0.5$ is



α - Cut of an Octagonal Fuzzy Number

The α - cut of a normal octagonal fuzzy number is given by

$$[\tilde{A}]_\alpha = \begin{cases} \left[a_1 + \frac{\alpha}{k}(a_2 - a_1),\quad a_8 - \frac{\alpha}{k}(a_8 - a_7)\right] & for \quad \alpha \in [0,k] \\ \left[a_3 + \frac{\alpha - k}{1-k}(a_4 - a_3),\quad a_6 - \frac{\alpha - k}{1-k}(a_6 - a_5)\right] & for \quad \alpha \in [k,1] \end{cases}$$

Signed Distance Method

Defuzzification of $\tilde{A}$ can be found by signed distance method. If $\tilde{A}$ is a octagonal fuzzy number then the signed distance from $\tilde{A}$ to 0 is defined as

$$d(\tilde{A},\tilde{0}) = \frac{1}{2}\int_0^1 ([A_L(\alpha), A_R(\alpha)],\tilde{0})d\alpha$$

where

$$A_\alpha = [A_L(\alpha), A_R(\alpha)]$$

$$A_\alpha = [a + (b-a)\alpha, d - (d-c)\alpha], \alpha \in [0,1]$$

Notations

A – Ordering cost per order

$\tilde{A}$ - Fuzzy Ordering cost per order

H – Holding cost per unit quantity per unit time

$\tilde{H}$ - Fuzzy holding cost per order

T – Length of the plan

D – Demand with time period [0, T]

Q – Order quantity per cycle

$\tilde{Q}$ - Fuzzy Order quantity per cycle

TC – Total cost for the period [0, T]

$T\tilde{C}$ - Fuzzy total cost for the period [0, T]

Assumptions

- The inventory system involves only one item.
- The replenishment rate is infinite.
- Demand is constant.
- Shortages are not allowed.
- Ordering cost, holding cost and order quantity are fuzzy in nature.

Mathematical Formulation of a Fuzzy Inventory Problem

Total cost of the inventory problem is obtained by the following equation:

$$T\tilde{C} = \left[\tilde{H}\times\frac{T\tilde{Q}}{2}\right]+\left[\tilde{A}\times\frac{D}{\tilde{Q}}\right] \tag{1}$$

Substituting octagonal fuzzy numbers for holding cost and ordering cost in the above equation,

$$= \left[(h_1,h_2,h_3,h_4,h_5,h_6,h_7,h_8)\times\frac{T\tilde{Q}}{2}\right]+\left[(a_1,a_2,a_3,a_4,a_5,a_6,a_7,a_8)\times\frac{D}{\tilde{Q}}\right] \tag{2}$$

Simplifying the above equation,

$$T\tilde{C} = \left[h_1\frac{T\tilde{Q}}{2},h_2\frac{T\tilde{Q}}{2},h_3\frac{T\tilde{Q}}{2},h_4\frac{T\tilde{Q}}{2},h_5\frac{T\tilde{Q}}{2},h_6\frac{T\tilde{Q}}{2},h_7\frac{T\tilde{Q}}{2},h_8\frac{T\tilde{Q}}{2}\right]$$
$$+\left[a_1\frac{D}{\tilde{Q}},a_2\frac{D}{\tilde{Q}},a_3\frac{D}{\tilde{Q}},a_4\frac{D}{\tilde{Q}},a_5\frac{D}{\tilde{Q}},a_6\frac{D}{\tilde{Q}},a_7\frac{D}{\tilde{Q}},a_8\frac{D}{\tilde{Q}}\right]$$

$$=\left[\begin{array}{l} h_1\frac{T\tilde{Q}}{2}+a_1\frac{D}{\tilde{Q}},h_2\frac{T\tilde{Q}}{2}+a_2\frac{D}{\tilde{Q}},h_3\frac{T\tilde{Q}}{2}+a_3\frac{D}{\tilde{Q}},h_4\frac{T\tilde{Q}}{2} \\ +a_4\frac{D}{\tilde{Q}},h_5\frac{T\tilde{Q}}{2}+a_5\frac{D}{\tilde{Q}},h_6\frac{T\tilde{Q}}{2}+a_6\frac{D}{\tilde{Q}},h_7\frac{T\tilde{Q}}{2}+a_7\frac{D}{\tilde{Q}},h_8\frac{T\tilde{Q}}{2}+a_8\frac{D}{\tilde{Q}}\end{array}\right]$$

Finding the left and right α - cut for the equation,

$$L(\alpha)=\frac{T\tilde{Q}}{2}\left[h_1+\frac{\alpha}{k}(h_2-h_1)\right]+\frac{D}{\tilde{Q}}\left[a_1+\frac{\alpha}{k}(a_2-a_1)\right],\frac{T\tilde{Q}}{2}\left[h_8-\frac{\alpha}{k}(h_8-h_7)\right]+\frac{D}{\tilde{Q}}\left[a_8-\frac{\alpha}{k}(a_8-a_7)\right]$$

$$R(\alpha)=\frac{T\tilde{Q}}{2}\left[h_3+\frac{\alpha-k}{1-k}(h_4-h_3)\right]+\frac{D}{\tilde{Q}}\left[a_3+\frac{\alpha-k}{1-k}(a_4-a_3)\right],$$
$$\frac{T\tilde{Q}}{2}\left[h_6-\frac{\alpha-k}{1-k}(h_6-h_5)\right]+\frac{D}{\tilde{Q}}\left[a_6-\frac{\alpha-k}{1-k}(a_6-a_5)\right]$$

Defuzzifying $T\tilde{C}$ by using signed distance method we get,

$$d(T\tilde{C}(\tilde{A},\tilde{H}),0)=\frac{1}{2}\int_0^1[A_L(\alpha)+A_R(\alpha)]d\alpha$$

$$=\frac{1}{2}\int_0^1\left\{\begin{array}{l}\left[\frac{T\tilde{Q}}{2}\left[h_1+\frac{\alpha}{k}(h_2-h_1)+h_3+\frac{\alpha-k}{1-k}(h_4-h_3)\right]+\frac{D}{\tilde{Q}}\left[a_1+\frac{\alpha}{k}(a_2-a_1)+a_3+\frac{\alpha-k}{1-k}(a_4-a_3)\right]\right],\\ \left[\frac{T\tilde{Q}}{2}\left[h_8-\frac{\alpha}{k}(h_8-h_7)+h_6-\frac{\alpha-k}{1-k}(h_6-h_5)\right]+\frac{D}{\tilde{Q}}\left[a_8-\frac{\alpha}{k}(a_8-a_7)+a_6-\frac{\alpha-k}{1-k}(a_6-a_5)\right]\right]\end{array}\right\}d\alpha$$

Integrating and simplifying we get,

$$\begin{array}{l}=\frac{T\tilde{Q}}{4}\left\{\left[h_1+\frac{1}{2k}(h_2-h_1)+h_3+\frac{1-2k}{2(1-k)}(h_4-h_3)\right],\left[h_8-\frac{1}{2k}(h_8-h_7)+h_6-\frac{1-2k}{2(1-k)}(h_6-h_5)\right]\right\}\\ +\frac{D}{\tilde{Q}}\left\{\left[a_1+\frac{1}{2k}(a_2-a_1)+a_3+\frac{1-2k}{2(1-k)}(a_4-a_3)\right],\left[a_8-\frac{1}{2k}(a_8-a_7)+a_6-\frac{1-2k}{2(1-k)}(a_6-a_5)\right]\right\}\end{array} \quad (3)$$

= F (Q) (say)

Case 1: k = 0.5

Then Equation (3) becomes

$$F(Q)=\frac{T\tilde{Q}}{4}\{(h_2+h_3),(h_6+h_7)\}+\frac{D}{\tilde{Q}}\{(a_2+a_3),(a_6+a_7)\}$$

F(Q) is minimum when $\frac{dF(Q)}{dQ}=0$ and $\frac{d^2F(Q)}{dQ^2}>0$

Now, $\frac{dF(Q)}{dQ}=0$ gives the economic order quantity as:

$$\tilde{Q}_D=\sqrt{\frac{4D[(a_2+a_3),(a_6+a_7)]}{T[(h_2+h_3),(h_6+h_7)]}} \quad (4)$$

Also, at Q = Q_D, we have $\frac{d^2F(Q)}{dQ^2} > 0$

This shows that F(Q) is minimum at Q = Q_D.

And from the equation (3),

$$F(Q) = \frac{T\tilde{Q}_D}{4}\{(h_2+h_3),(h_6+h_7)\} + \frac{D}{\tilde{Q}_D}\{(a_2+a_3),(a_6+a_7)\} \quad (5)$$

Case 2: k = 0.3 (0 < k < 0.5)

Then Equation (3) becomes

$$\begin{aligned} F(Q) = \frac{T\tilde{Q}}{4}&\{[-0.67h_1+1.67h_2+0.71h_3+0.29h_4],[-0.67h_8+1.67h_7+0.71h_6+0.29h_5]\} \\ +\frac{D}{\tilde{Q}}&\{[-0.67a_1+1.67a_2+0.71a_3+0.29a_4],[-0.67a_8+1.67a_7+0.71a_6+0.29a_5]\} \end{aligned} \quad (6)$$

F (Q) is minimum when $\frac{dF(Q)}{dQ} = 0$ and $\frac{d^2F(Q)}{dQ^2} > 0$

Now, $\frac{dF(Q)}{dQ} = 0$ gives the economic order quantity as:

$$\tilde{Q}_D = \sqrt{\frac{4D\{[-0.67a_1+1.67a_2+0.71a_3+0.29a_4],[-0.67a_8+1.67a_7+0.71a_6+0.29a_5]\}}{T\{[-0.67h_1+1.67h_2+0.71h_3+0.29h_4],[-0.67h_8+1.67h_7+0.71h_6+0.29h_5]\}}} \quad (7)$$

Also, at Q = Q_D, we have $\frac{d^2F(Q)}{dQ^2} > 0$

This shows that F(Q) is minimum at Q = Q_D.

And from equation (3),

$$\begin{aligned} F(Q) = \frac{T\tilde{Q}_D}{4}&\{[-0.67h_1+1.67h_2+0.71h_3+0.29h_4],[-0.67h_8+1.67h_7+0.71h_6+0.29h_5]\} \\ +\frac{D}{\tilde{Q}_D}&\{[-0.67a_1+1.67a_2+0.71a_3+0.29a_4],[-0.67a_8+1.67a_7+0.71a_6+0.29a_5]\} \end{aligned} \quad (8)$$

Case 3: k = 0.7 (0.5 < k < 1)

Then Equation (3) becomes

$$F(Q)=\frac{T\tilde{Q}}{4}\{[0.29h_1+0.71h_2+1.67h_3-0.67h_4],[0.29h_8+0.71h_7+1.67h_6-0.67h_5]\}$$
$$+\frac{D}{\tilde{Q}}\{[0.29a_1+0.71a_2+1.67a_3-0.67a_4],[0.29a_8+0.71a_7+1.67a_6-0.67a_5]\} \quad (9)$$

F(Q) is minimum when $\frac{dF(Q)}{dQ}=0$ and $\frac{d^2F(Q)}{dQ^2}>0$

Now, $\frac{dF(Q)}{dQ}=0$ gives the economic order quantity as:

$$\tilde{Q}_D=\sqrt{\frac{4D\{[0.29a_1+0.71a_2+1.67a_3-0.67a_4],[0.29a_8+0.71a_7+1.67a_6-0.67a_5]\}}{T\{[0.29h_1+0.71h_2+1.67h_3-0.67h_4],[0.29h_8+0.71h_7+1.67h_6-0.67h_5]\}}} \quad (10)$$

Also, at Q = Q_D, we have $\frac{d^2F(Q)}{dQ^2}>0$

This shows that F(Q) is minimum at Q = Q_D.

And from the equation (3),

$$F(Q)=\frac{T\tilde{Q}_D}{4}\{[0.29h_1+0.71h_2+1.67h_3-0.67h_4],[0.29h_8+0.71h_7+1.67h_6-0.67h_5]\}$$
$$+\frac{D}{\tilde{Q}_D}\{[0.29a_1+0.71a_2+1.67a_3-0.67a_4],[0.29a_8+0.71a_7+1.67a_6-0.67a_5]\} \quad (11)$$

Numerical Examples

Let T = 6, H = (8, 10, 11, 12, 13, 14, 15, 16), A = (15, 17, 19, 21, 23, 24, 25, 27)

Then

Table 1

D	K=0.5		K=0.3		K=0.7	
	Q	TC	Q	TC	Q	TC
450	22.68	1428.71	22.51	1522.16	22.43	1348.22
500	27.88	1523.7	26.41	1614.32	29.12	1447.83
550	24.89	1579.54	24.89	1682.82	24.79	1490.52
600	25.99	1649.78	25.99	1757.64	25.90	1556.79
650	27.06	1717.14	27.06	1829.41	26.96	1620.35

CONCLUSIONS

In this paper we introduced a new method of solving a fuzzy inventory problem involving octagonal fuzzy numbers. Triangular fuzzy numbers and trapezoidal fuzzy numbers were used in the earlier fuzzy inventory models developed by different authors. We have taken ordering cost, holding cost and order quantity as octagonal fuzzy numbers. We considered constant demand during the time period T. Shortage was not taken into account. Numerical examples were given to evaluate the proposed model.

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